4740
Fall 2024
Test 1 Solutions

① (a)

$$\underset{25}{\underset{\substack{\text{not} \\ Z}}{\rule{1.5cm}{0.4pt}}} \cdot \underset{26}{\underset{\text{letter}}{\rule{1.5cm}{0.4pt}}} \cdot \underset{10}{\underset{\#}{\rule{1cm}{0.4pt}}} \cdot \underset{10}{\underset{\#}{\rule{1cm}{0.4pt}}} \cdot \underset{10}{\underset{\#}{\rule{1cm}{0.4pt}}} \cdot \underset{10}{\underset{\#}{\rule{1cm}{0.4pt}}} \cdot \underset{2}{\underset{\square/\!\!\star}{\rule{1cm}{0.4pt}}} \cdot \underset{2}{\underset{\square/\!\!\star}{\rule{1cm}{0.4pt}}}$$
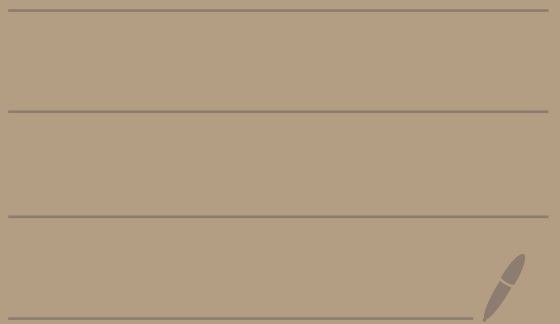
$$\boxed{26,000,000 \text{ license plates}}$$

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① (b)

$$\underset{5}{\underset{\uparrow}{\rule{1.5cm}{0.4pt}}} \quad \overset{5}{\rule{1.5cm}{0.4pt}} \quad \underset{4}{\underset{\uparrow}{\rule{1.5cm}{0.4pt}}} \cdot \underset{3}{\underset{\uparrow}{\rule{1.5cm}{0.4pt}}} \cdot \underset{2}{\underset{\uparrow}{\rule{1.5cm}{0.4pt}}} \cdot \underset{1}{\underset{\uparrow}{\rule{1.5cm}{0.4pt}}}$$

\# such permutations is

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$$

② (a) $S = \{ (1,1), (1,2), (1,3),$
$(2,1), (2,2), (2,3),$
$(3,1), (3,2), (3,3) \}$

(b)
$E = \{ (1,1), (1,2), (1,3) \}$
$F = \{ (1,2), (2,2), (3,2) \}$
$E \cap F = \{ (1,2) \}$
$E \cup F = \{ (1,1), (1,2), (1,3), (2,2), (3,2) \}$
$\overline{F} = \{ (1,1), (1,3), (2,1), (2,3), (3,1), (3,3) \}$

(c)
$P(E) = \dfrac{3}{9} = \dfrac{1}{3}$

$P(F) = \dfrac{3}{9} = \dfrac{1}{3}$

③ (a)

HW 2 #9(a)
Practice test #3(a)
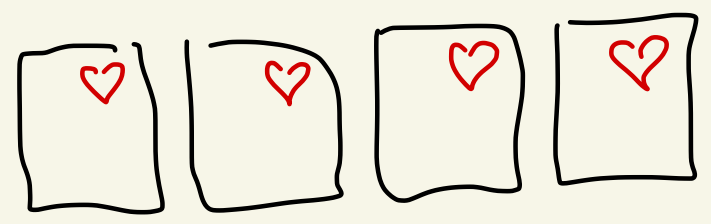
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③ (b)

HW 2 #12(a)

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④ (a)

$$\binom{52}{4} = \frac{52!}{4! \, 48!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48!}{4! \, 48!}$$

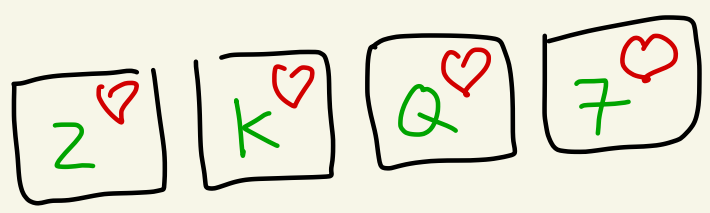$$= \frac{6,497,400}{24} = \boxed{270,725}$$

Step 1: Choose the suit

4 ways

♡ ♡ ♡ ♡

Step 2: Choose the 4 face values

$$\binom{13}{4} = \frac{13!}{4! \, 9!} = \frac{13 \cdot 12 \cdot 11 \cdot 10}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 715 \text{ ways}$$

$2♡$ $K♡$ $Q♡$ $7♡$

$$\text{Probability} = \frac{(4)(715)}{270,725} = \frac{2,860}{270,725} \approx \boxed{0.01056\ldots}$$
$$\approx \boxed{1.056\%}$$

⑤

(a) $\binom{12}{3} = \dfrac{12!}{3! \, 9!} = \dfrac{12 \cdot 11 \cdot 10}{3!} = \boxed{220}$

(b) $\dfrac{\binom{6}{3}}{\binom{12}{3}} = \dfrac{12}{220} \approx 0.0545\ldots \approx \boxed{5.45\%}$

(c) $\dfrac{\binom{6}{1}\binom{6}{2}}{\binom{12}{3}} = \dfrac{6 \cdot 15}{220} = \dfrac{90}{220} \approx 0.409\ldots$

$\approx \boxed{40.9\%}$